Mass Varying Neutrinos With More Than One Species Of Neutrinos

Ole Eggers Bjælde

Department of Physics and Astronomy, University of Aarhus, Ny Munkegade, Bld. 1520, DK-8000 Aarhus C.

Abstract. In the context of Mass Varying Neutrinos(MaVaNs) we study a model in which a scalar field is coupled to more than one species of neutrinos with different masses. In general, adiabatic models of non-relativistic MaVaNs are heavily constrained by their stability towards the formation of neutrino nuggets. These constraints also apply to models with more than one neutrino species, and we find that using the lightest neutrino, which is still relativistic, as an explanation for dark energy does not work because of a feedback mechanism from the heavier neutrinos.

Keywords: Dark Energy,Neutrinos
PACS: 95.36.+x, 98.80.-k

1. INTRODUCTION

Precise observations of the cosmic microwave background [1, 2], the large scale structure of galaxies [3], and distant type Ia supernovae [4] have led to a standard model of cosmology in which the energy density is dominated by dark energy with negative pressure, leading to an accelerated expansion of the universe.

A proposal to explain dark energy is the so-called mass varying neutrino (MaVaN) model [5, 6, 7] in which a light scalar field couples to neutrinos, see also [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28].

In this paper we discuss the suggestion that the lightest neutrino which can be relativistic today may be responsible for dark energy. We find that there is some evidence that the relativistic neutrino will feel an instability towards the formation of neutrino nuggets.

In the next section we briefly review the formalism needed to study mass varying neutrinos, in section 3 we discuss MaVaNs with a relativistic neutrino, and in section 4 we conclude.

2. MASS VARYING NEUTRINOS

In the MaVaN model [5, 6, 7] we introduce a coupling between neutrinos and a light scalar field, and the coupled fluid then acts as dark energy. In this way, the neutrino mass m_ν is generated from the vacuum expectation value (VEV) of the scalar field. The effective potential is defined by

$$V(\phi) = V_\phi(\phi) + (\rho_\nu - 3P_\nu) \qquad (1)$$

where $V_\phi(\phi)$ is the scalar field potential, a is the scale factor, $\rho_\nu(m_\nu(\phi),a)$ is the neutrino energy density, and $P_\nu(m_\nu(\phi),a)$ is the neutrino pressure.

The energy density and pressure of the scalar field are given by the usual expressions,

$$\rho_\phi(a) = \frac{1}{2a^2}\dot{\phi}^2 + V_\phi(\phi) \quad \text{and} \quad P_\phi(a) = \frac{1}{2a^2}\dot{\phi}^2 - V_\phi(\phi). \tag{2}$$

Defining $w = P_{\rm DE}/\rho_{\rm DE}$ to be the equation of state of the coupled dark energy fluid, where $P_{\rm DE} = P_\nu + P_\phi$ denotes its pressure and $\rho_{\rm DE} = \rho_\nu + \rho_\phi$ its energy density, the requirement of energy conservation gives,

$$\dot{\rho}_{DE} + 3H\rho_{DE}(1+w) = 0, \tag{3}$$

where $H \equiv \frac{\dot{a}}{a}$ and dots to refer to the derivative with respect to conformal time. Combining with Eq. (3), one arrives at a modified Klein-Gordon equation describing the evolution of ϕ,

$$\ddot{\phi} + 2H\dot{\phi} + a^2 V'_\phi = -a^2\beta(\rho_\nu - 3P_\nu), \tag{4}$$

where primes denote derivatives with respect to ϕ ($' = \partial/\partial\phi$) and $\beta = \frac{d\log m_\nu}{d\phi}$ is the coupling between the scalar field and the neutrinos.

It can be quite instructive to look at the behavior of MaVaN models in the case of non-relativistic neutrinos $P_\nu \simeq 0$, such that Eq. (1) takes the form

$$V = \rho_\nu + V_\phi \tag{5}$$

Naturalness suggests we pick a scalar field mass(Curvature scale of the potential) to be much larger than the expansion rate of the Universe,

$$V'' = \rho_\nu\left(\beta' + \beta^2\right) + V''_\phi \equiv m_\phi^2 \gg H^2. \tag{6}$$

In this case, the adiabatic solution to the Klein-Gordon Eq. (4) applies [7]. As a consequence, the scalar field will sit in the minimum of its effective potential V at all times

$$V' = \rho'_\nu + V'_\phi = \beta\rho_\nu + V'_\phi = 0 \tag{7}$$

MaVaNs models can become unstable on sub-Hubble scales $m_\phi^{-1} < a/k < H^{-1}$ in the non-relativistic regime of the neutrinos, where the perturbations $\delta\rho_\nu$ evolve adiabatically.

In Ref. [18](see also [30, 31, 32, 33, 34]) it is shown that the equation of motion for the neutrino density contrast $\frac{\delta\rho_\nu}{\rho_\nu}$ in the regime $m_\phi^{-1} < a/k < H^{-1}$ can be written as

$$\ddot{\delta}_\nu + H\dot{\delta}_\nu + \left(\frac{\delta p_\nu}{\delta\rho_\nu}k^2 - \frac{3}{2}H^2\Omega_\nu\frac{G_{\rm eff}}{G}\right)\delta_\nu = \frac{3}{2}H^2\left[\ \Omega_{\rm CDM}\delta_{\rm CDM} + \Omega_b\delta_b\ \right] \tag{8}$$

where

$$G_{\rm eff} = G\left(1 + \frac{2\beta^2 M_{\rm pl}^2}{1 + \frac{a^2}{k^2}\{V''_\phi + \rho_\nu\beta'\}}\right) \quad \text{and} \quad \Omega_i = \frac{a^2\rho_i}{3H^2 M_{\rm pl}^2}. \tag{9}$$

Since neutrinos interact through gravity as well as through the force mediated by the scalar field, they feel an effective Newton's constant $G_{\rm eff}$ as defined in Eq. (9). The force depends upon the MaVaN model specific functions β and V_ϕ and takes values between G and $G(1+2\beta^2 M_{\rm pl}^2)$ on very large and small length scales, respectively.

In certain cases of strong coupling neutrinos suffer an instability towards clumping in which case they stop behaving as dark energy [11]. In Ref. [18] a criterion for the stability was developed. $\left(1+\frac{2\beta^2 M_{\rm pl}^2}{1+\frac{a^2}{k^2}\{V_\phi''+\rho_\nu\beta'\}}\right)\Omega_\nu\delta_\nu < \Omega_{\rm CDM}\delta_{\rm CDM}+\Omega_b\delta_b$. This can be recast in a more convenient form $\frac{2\beta^2 M_{\rm pl}^2}{1+\frac{a^2}{k^2}\{V_\phi''+\rho_\nu\beta'\}}\Omega_\nu < \Omega_{\rm M}$, where we have neglected the effect of baryons compared to cold dark matter and we have assumed the density contrasts of roughly the same order.

From the considerations above one can establish a list of criteria that MaVaN models need to fulfill. This was done in Ref. [29] where it was stressed that for single field MaVaN models that satisfy adiabaticity, the right amount dark energy today, correct neutrino mass as well as stability cannot be simultaneously fulfilled. This has previously been stated by Refs. [11] and [18].

Hence it has been suggested, in the context of multiple scalar field models, that neutrinos may be stable towards clustering if our effective potential has two minima: A false minimum in which our universe sits and a true minimum. The offset between the two minima is then interpreted as the dark energy density. The model is implemented in SUSY to avoid problems with small scalar field masses. Stability is ensured by letting the lightest relativistic neutrino be responsible for the dark energy [16]. Below we analyze this suggestion.

3. MAVAN MODEL WITH A RELATIVISTIC NEUTRINO

We assume the scalar field couples to all light neutrino species for naturalness reasons. In the case of three hierarchical neutrino masses, one would naively assume that, as a result of the coupling, the two heavier neutrinos would become unstable to clustering, whereas the lightest would remain stable. In the following we will argue that this is not possible, since a feedback from the growth of the heavy neutrino perturbations will cause the relativistic neutrino perturbation to grow as well. The following equation(see Ref. [13]) explains this

$$\delta\rho_\nu = \frac{1}{a^4}\int q^2 dq d\Omega \varepsilon f_0(q)\Psi + \delta\phi\beta(\rho_\nu - 3P_\nu), \tag{10}$$

where $\varepsilon^2 = q^2 + m_\nu^2 a^2$ and f_0 is the unperturbed Fermi-Dirac distribution with a small perturbation Ψ. The equation explains the growth of the neutrino perturbation and applies to both the interaction between a relativistic neutrino and a scalar field as well as that of a non-relativistic neutrino and a scalar field.

We consider a system consisting of two heavy neutrinos and one relativistic neutrino each interacting with the same scalar field. The following list of events will take place. In the beginning, the relativistic neutrino will not feel the presence of the heavier ones.

However, the heavy neutrinos will feel the coupling which will drive their perturbations $\delta\rho_\nu(\text{heavy})$ to large values. In this way, the heavy neutrinos start clumping.

As was demonstrated in Ref. [18], the scalar field perturbation is effectively proportional to the neutrino perturbation for the interaction with heavy neutrinos. This means that since $\delta\rho_\nu(\text{heavy})$ is growing, $\delta\phi$ will also grow accordingly.

Regarding the relativistic neutrinos, the perturbations $\delta\rho_\nu(\text{rel})$ will grow as indicated in Eq. 10. The second term in the equation consists of three important contributions, firstly $\delta\phi$ which is growing (this is the same $\delta\phi$ as listed above since we only have one scalar field). Secondly we have a coupling which we for simplicity assume to be constant (in reality this will be a growing quantity for most cases). And finally we have $(\rho_\nu - 3P_\nu)$ which is a suppression factor of the order m/E - this factor will act to delay the growth of $\delta\rho_\nu(\text{rel})$. However, since $\delta\phi$ will continue its growth, the inevitable conclusion is that $\delta\rho_\nu(\text{rel})$ will eventually start to grow. Hence, there exists a type of feedback mechanisms between the heavy and the relativistic neutrinos. One could of course argue that we are exactly living in a transition regime when $\delta\rho_\nu(\text{rel})$ has still not turned unstable. However, that would require serious fine-tuning.

A graphical illustration of the example above is given in Fig. 1. This is done in the framework of a model with a Coleman-Weinberg type scalar field potential similar to the one presented in Refs.[7] and [18].

$$V_\phi = V_0 \log\left(1 + k^2\phi^2\right) \tag{11}$$

In order to avoid possible pathological behaviour, we choose a mass term slightly different than the one in Ref. [18], namely one that does not become infinite when the VEV of the scalar field goes to zero. However, it still behaves as $1/\phi$ for small ϕ.

$$m_\nu = -\frac{1}{2}\lambda\phi + \sqrt{\frac{1}{4}\lambda^2\phi^2 + m_d^2}, \tag{12}$$

which can be derived from solving the mass matrix $(\lambda\phi, m_d; m_d, 0)$.

What happens is shown in Fig. 1, where we can see that for the higher redshifts the density contrast of heavier neutrinos behaves moderately as predicted by GR. The cdm term in eq. 8 sources the slow growth of these heavy neutrinos. As their masses increase more and more the coupling term slowly takes over and becomes the dominant term in eq. 8. Eventually this leads to the unstable growth of their density contrast. What happens next is that once the growth of the heavy neutrinos enter the quasi-linear regime, immediately the effect can be seen on the growth of the relativistic neutrino density contrast. This starts blowing up, and a short while later the system of equations we are solving effectively breaks down, which can be seen by the unnatural strong growth of the cdm density contrast.

This gives us a hint that as a result of a feedback mechanism, the fast growing behavior of the heavy neutrino density contrast causes the relativistic neutrinos to start clumping as well.

Hence the neutrino scalar-field fluid will start acting as a cold dark matter component (clustering neutrinos) and hence cannot be attributed to dark energy.

Now, one can of course argue that the rise in the density contrast of the relativistic neutrino species only happens once the heavier ones have turned non-linear. In this

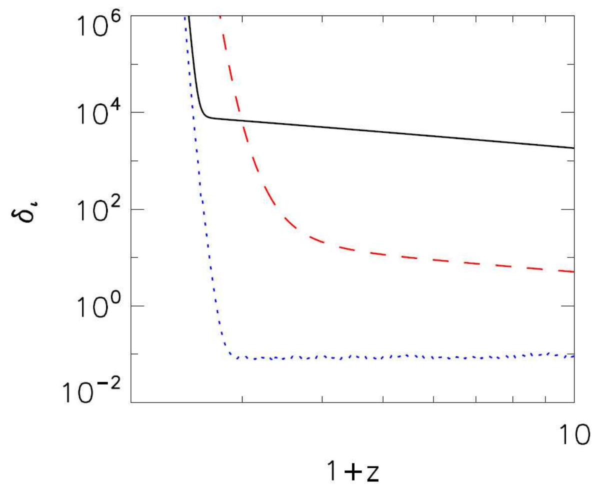

FIGURE 1. Density contrasts plotted as a function of redshift for a system consisting of one light and two heavy MaVaN neutrinos each interacting with the same scalar field. The scale is $k = 0.1\,\mathrm{Mpc}^{-1}$ and we choose the current neutrino masses $m_\nu(\mathrm{rel}) = 10^{-7}$ eV and $m_\nu(\mathrm{heavy}) = 0.3\,\mathrm{eV}$ (Note that the choice of current neutrino masses does not affect the result qualitatively). The solid line is cdm-density contrast, the dotted line is the light neutrino density contrast, and the dashed line is the heavy neutrino density contrast. The heavy neutrinos grow essentially as cdm until the coupling becomes large enough for the instabilities to set in. The light neutrino is still relativistic, and its density contrast oscillates as acoustic waves. However, due to a feedback mechanism, the relativistic neutrino density contrast tracks that of the non-relativistic neutrino around the time the growth of the heavy neutrino perturbations become quasi-linear. I.e. both neutrino species will clump. Note that the cdm perturbations also blow up at late times. This is an effect of the system of differential equations breaking down as all parameters go to infinity.

regime the linear code does not apply. However, from a close-up look at the data, we emphasize that the rise of the density contrast happens in the quasi-linear regime of the heavy neutrino perturbations, where the code does still apply.

The reason that the relativistic neutrino is able to clump is that it will acquire an effective mass, thus it cannot be regarded as a relativistic particle. Unfortunately, we cannot use conventional bounds to constrain this effect for the following reason: Once the evolution of the non-relativistic neutrinos becomes non-linear, the whole system of equations we are solving, starting with the modified KG equation breaks down. This has the effect that all current bounds are no longer valid, as these are established in the linear regime.

4. CONCLUSION

Single scalar field models can be used to explain late-time acceleration in the MaVaN scenario. However, in general using these potentials leads to instabilities towards neutrino bound states unless certain criteria are relaxed.

Accordingly it has been suggested to include an extra scalar field in the treatment. This has some very nice features and is easily capable of obtaining late-time acceleration as well as $\Omega_{\mathrm{DE}} = 0.7$ today. However, one drawback is the need for the lightest neutrino to

be relativistic today. As was explained above, the feedback mechanism will eventually cause the relativistic neutrino to start clustering and hence the coupled fluid will cease to act as dark energy.

ACKNOWLEDGMENTS

OEB would like to thank Steen Hannestad for great supervision, and Lily Schrempp, Carsten van de Bruck, Anthony Brookfield, David Mota, and Domenico Tocchini-Valentini for collaboration. OEB acknowledge the use of the publicly available CMB-FAST code [35].

This is a contribution to the conference proceedings from the 4th International Workshop on the Dark Side of the Universe 2008 in Cairo and will appear in AIP Conference Proceedings 1115 [36].

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